Profinite and Finite Groups Associated with Loop and Diffeomorphism Groups of Non-Archimedean Manifolds. *

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1 Introduction.

The importance of such groups in the non-Archimedean functional analysis, representation theory and mathematical physics is clear and also can be found in the references given below [2, 9, 10, 14, 15, 17, 18]. This article is devoted to one aspect of such groups: their structure from the point of view of the p-adic compactficiation (see also about Banaschewski compactification in [17]). This also opens new possibilites for studying their representations as restrictions of representations of p-adic compactifications, which are constructed below such that they also are groups.

At first we remind basic facts and notations, which are given in detail in references [9, 10, 11, 17, 19]. For a diffeomorphism group $Diff(M)$ of a Banach manifold over a local field $\mathbf{K}$ there are clopen subgroups W such that they contain a sequence of profinite subgroups G_n with $G_n \subset G_{n+1}$ for each $n \in \mathbf{N}$ and $\bigcup_n G_n$ is dense in W. A loop group $L_t(M,N)$ is defined as a quotient space of a family of mappings $f: M \to N$ of class C^t of one Banach manifold M into another N over the same local field $\mathbf{K}$ such that $\lim_{z\to s}(\bar{\Phi}^v f)(z;h_1,...,h_n;\zeta_1,...,\zeta_n) = 0$ for each $0 \le v \le t$, where M and N are embedded into the corresponding Banach spaces X and

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Y, $cl(M) = M \cup \{s\}$, $cl(M)$ and N are clopen in X and Y respectively, $0 \in N$, $(\bar{\Phi}^v f)(z; h_1, ..., h_n; \zeta_1, .., \zeta_n)$ are continuous extensions of difference quotients, $z \in M$, $h_1, ..., h_n$ are nonzero vectors in X, $\zeta_1, ..., \zeta_n \in \mathbf{K}$ such that $z + \zeta_1 h_1 + ... + \zeta_n h_n \in M$, $n = [v] + sign\{v\}$. p-adic completions of clopen subgroups W of loop groups G and diffeomorphism groups G are considered. In the case of the diffeomorphism group the p-adic completion produces weakened topology on W relative to which it remains a topological group. In the case of the loop group the p-adic completion produces a new topological group V in which the initial group W is embedded as a dense subgroup such that $V \ne W$. The topology on W inherited from V is weaker than the initial one. For the compact manifold M in the case of the diffeomorphism group the p-adic completion of W produces profinite group. For the locally compact manifolds M and N in the case of the loop group $L^t(M, N)$ the p-adic completion of W produces its embedding into $\mathbf{Q_p}^{\mathbf{N}}$. When W is bounded relative to the corresponding metric in $L^t(M, N)$, then W is embedded into $\mathbf{Z_p}^{\mathbf{N}}$. The group $Diff(M)$ is perfect and simple, on the other hand, the group $L^t(M, N)$ is commutative. The notation given below and the corresponding definitions are given in detail in [9, 11].

2 p-Adic Completion of Diffeomorphism Groups.

2.1. Notations and Remarks. Let N be a compact manifold over a local field $\mathbf{K}$, that is, $\mathbf{K}$ is a finite algebraic extension of the field of p-adic numbers $\mathbf{Q_p}$ [20]. Let also N be embedded into $B(\mathbf{K}^{\mathsf{n}}, 0, 1)$ as a clopen (closed and open at the same time) subset [4, 13], where $\mathsf{n} \in \mathbf{N}$, $B(X, y, r) := \{z : z \in X; d_X(y, z) \le r\}$ denotes a clopen ball in a space X with an ultramentric d_X. The ball $B(\mathbf{K}^{\mathsf{n}}, 0, 1)$ has the ring structure with coordinatewise addition and multiplication, in particular $B(\mathbf{Q_p}, 0, 1) = \mathbf{Z_p}$ is the ring of entire p-adic numbers. The ring $B(\mathbf{K}^{\mathsf{n}}, 0, 1)$ is homeomorphic with the projective limit $B(\mathbf{K}^{\mathsf{n}}, 0, 1) = pr - \lim_k \mathbf{S_{p^k}}^{\mathsf{n}}$, $\mathbf{S_{p^k}}$ is a finite ring consisting of p^{kc} elements such that $\mathbf{S_{p^k}}$ is equal to the quotient ring $B(\mathbf{K}, 0, 1)/B(\mathbf{K}, 0, p^{-k})$, $\mathbf{S_{p^k}}^{\mathsf{n}}$ is a product of n copies of $\mathbf{S_{p^k}}$, c is a dimension $dim_{\mathbf{Q_p}}\mathbf{K}$ of $\mathbf{K}$ over $\mathbf{Q_p}$ as a $\mathbf{Q_p}$-linear space. In particular $B(\mathbf{Q_p}, 0, 1)/B(\mathbf{Q_p}, 0, p^{-k}) = \mathbf{Z_p}/p^k\mathbf{Z_p} = \mathbf{F_{p^k}}$ is a finite ring consisting of p^k elements, $\mathbf{Z_p}$ is the ring of p-adic integer numbers, $aB := \{x : x = ab, b \in B\}$ for a multiplicative group B and its element $a \in B$, $k \in \mathbf{N}$ [17, 20]. For each $m \ge k$ there are the following quotient mappings

(ring homomorphisms): $\pi_m : B(\mathbf{K}, 0, 1) \to \mathbf{S}_{\mathbf{p}^\mathsf{m}}$ and $\pi_k^m : \mathbf{S}_{\mathbf{p}^\mathsf{m}} \to \mathbf{S}_{\mathbf{p}^\mathsf{k}}$. This induces the quotient mappings $\pi_m : N \to N_m$ and $\pi_k^m : N_m \to N_k$, where $N_m \subset \mathbf{S}_{\mathbf{p}^\mathsf{m}}$, $\pi_k^m \circ \pi_m = \pi_k$.

Let now M and N be two analytic compact manifolds embedded into $B(\mathbf{K}^\mathsf{m}, 0, 1)$ and $B(\mathbf{K}^\mathsf{n}, 0, 1)$ respectively as clopen subsets and $f \in C^t(M, N)$, where $C^t(M, N)$ denotes the space of functions $f : M \to N$ of class C^t, $t \geq 0$. For an integer t it is a space of t-times continuously differetiable functions in the sence of partial difference quotients (see in detail: [9, 11, 19]). Then $f = pr - \lim_k f_k$, where $f_k := \pi_k \circ f$. We inroduce the following notation: $C^t(M, N_k) := \pi_k \circ C^t(M, N) = \{f_k : f \in C^t(M, N)\}$, hence $C^t(M, N) = pr - \lim_k C^t(M, N_k)$ algebraically without taking into account topologies (or the limit of the inverse sequence, see §2.5 [6] and §§3.3, 12.202 [16]). Each function $f \in C^t(M, N)$ has a $C^t(B(\mathbf{K}^\mathsf{m}, 0, 1), \mathbf{K}^\mathsf{n})$-extension by zero on $B(\mathbf{K}^\mathsf{m}, 0, 1)$, hence it has the following decomposition $f = \sum_{l,m} f_m^l \bar{Q}_m e_l$ in the Amice's basis $\bar{Q}_m$, where e_l is the standard orthonormal basis in $\mathbf{K}^\mathsf{n}$ such that $e_l = (0, ..., 0, 1, 0, ...)$ with 1 in the l-th place, $m \in \mathbf{Z}^\mathsf{n}$, $m_l \geq 0$, $m = (m_1, ..., m_\mathsf{n})$, $f_m^l \in \mathbf{K}$ are expansion coefficients such that $\lim_{l+|m| \to \infty} |f_m^l|_\mathbf{K} J(t, m) = 0$, $\bar{Q}_m$ are polynomials on $B(\mathbf{K}^\mathsf{m}, 0, 1)$ with values in $\mathbf{K}$. The space $C^t(M, N)$ is supplied with the uniformity inherited from the Banach space $C^t(\mathbf{K}^\mathsf{m}, \mathbf{K}^\mathsf{n})$, $J(t, m) := \|\bar{Q}_m\|_{C^t(\mathbf{K}^\mathsf{m}, \mathbf{K})}$.

2.2. Lemma. *Each $f \in C^t(M, N)$ is a projective limit $f = pr - \lim_k f_k$ of polynomials $f_k = \sum_{l,m} f_{m,k}^l \bar{Q}_{m,k} e_l$ on rings $\mathbf{S}_{\mathbf{p}^\mathbf{k}}^\mathsf{m}$ with values in $\mathbf{S}_{\mathbf{p}^\mathbf{k}}^\mathsf{n}$, where $f_{m,k}^l \in \mathbf{S}_{\mathbf{p}^\mathsf{k}}$ and $\bar{Q}_{m,k}$ are polynomials on $\mathbf{S}_{\mathbf{p}^\mathbf{k}}^\mathsf{m}$ with values in $\mathbf{S}_{\mathbf{p}^\mathsf{k}}$.*

Proof. In view of §2.1 $f_k = \pi_k \circ f$ and $\pi_k \circ f(x) = \sum_{l,m} (\pi_k(f_m^l)(\pi_k \bar{Q}_m(x)) e_l$, since π_k is a ring homomorphism and $\pi_k(e_l) = e_l$. Then $\pi_k(a x^m) = a_k x^m(k)$ for each $a \in \mathbf{K}$ and $x \in B(\mathbf{K}^\mathsf{m}, 0, 1)$, where $x^m = x_1^{m_1} ... x_\mathsf{m}^{m_\mathsf{m}}$, $a_k = \pi_k(a)$ with $a_k \in \mathbf{S}_{\mathbf{p}^\mathsf{k}}$ and $x^m(k) = \pi_k(x^m)$ with $x(k) \in \mathbf{S}_{\mathbf{p}^\mathbf{k}}^\mathsf{m}$, consequently, $\pi_k(\bar{Q}_m(x)) = \bar{Q}_{m,k}(x(k))$. The series for f_k is finite, since $\pi_k(a) = 0$ for each $a \in \mathbf{K}$ with $|a| < p^{-k}$ and $\lim_{l+|m| \to \infty} |f_m^l|_\mathbf{K} J(t, m) = 0$.

2.3. Corolary. *The space $C^t(M, N_k)$ is isomorphic with the space $N_k^{M_k}$ of all mappings from M_k into N_k. Moreover, $(\mathbf{S}_{\mathbf{p}^\mathbf{k}}^\mathsf{n})^{(\mathbf{S}_{\mathbf{p}^\mathbf{k}}^\mathsf{m})}$ is a finite-dimensional space over the ring $\mathbf{S}_{\mathbf{p}^\mathsf{k}}$.*

Proof. From §2.2 it follows that there is only a finite number of $\mathbf{S}_{\mathbf{p}^\mathsf{k}}$-linearly independent polynomilas $\bar{Q}_{m,k}(x(k))$ (that is, in the module of the ring $\mathbf{S}_{\mathbf{p}^\mathsf{k}}$), since the rings $\mathbf{S}_{\mathbf{p}^\mathbf{k}}^\mathsf{m}$ and $\mathbf{S}_{\mathbf{p}^\mathsf{k}}$ are finite, also $z^a = z^b$ for each natural numbers a and b such that $a = b \ (mod \ (p^k))$ and each $z \in \mathbf{S}_{\mathbf{p}^\mathsf{k}}$. The space

$C^t(M, N_k)$ is isomorphic with $N_k^{M_k}$, since M_k and N_k are discrete.

2.4. Corollary. *$\pi_k \circ Diff^t(M)$ is isomorphic with the symmetric group $S_{\mathsf{n_k}}$, where $\mathsf{n_k}$ is the cardinality of M_k.*

Proof. If $h \in Diff^t(M)$, then $h_k(M_k) = M_k$, since $h(M) = M$. In view of §2.3 $\pi_k \circ Diff^t(M)$ is isomorphic with the following group $Hom(M_k)$ of all homeomorphisms h_k of M_k, that is, bijective surjective mappings $h_k : M_k \to M_k$. Using an enumeration of elements of M_k we get an isomorphism of $Hom(M_k)$ with $S_{\mathsf{n_k}}$.

2.5. Let $C_w(M, N) := pr - \lim_k N_k^{M_k}$ be a uniform space of continuous mappings $f : M \to N$ supplied with a uniformity inherited from products of uniform spaces $\prod_{k=1}^{\infty} N_k^{M_k}$ (see also §8.2 [6]). The spaces $C^t(M, N)$ and $C_w(M, N)$ are subsets of $\mathbf{K}$-linear spaces $C^t(M, \mathbf{K}^{\mathsf{n}})$ and $C^0(M, \mathbf{K}^{\mathsf{n}})$ respectively. We consider algebraic structures of subsets of the latter $\mathbf{K}$-linear spaces as inherited from them.

Corollary. *The space $C^t(M, N)$ is not algebraically isomorphic with $C_w(M, N)$, when $t > 0$. The uniform space $C_w(M, N)$ is uniformly isomorphic with $C^0(M, N)$, when the latter space is supplied with a weak uniformity inherited from $C^0(M, \mathbf{K}^{\mathsf{n}})$. The space $C_w(M, N)$ is compact.*

Proof. In view of §2.5 [6] we have that $C^0(M, N)$ and $C_w(M, N)$ coincide algebraically, since the connecting mappings π_n^m are uniformly continuous for each $m \geq n$. The space $C^0(M, \mathbf{K}^{\mathsf{n}})$ is $\mathbf{K}$-linear and its uniformity is completely defined by a neighbourhood base of zero. The set of all evaluation mappings in points of M produces the base of the topology of $C^0(M, \mathbf{K}^{\mathsf{n}})$. In its weak topology the latter space is isomorphic with the product $\prod_{x \in M} \mathbf{K}^{\mathsf{n}} = \mathbf{K}^{card(M)}$, since $card(M) = card(\mathbf{R}) = \mathsf{c}$. Then $C^0(M, N)$ and $C_w(M, N)$ have embeddings into $B(\mathbf{K}, 0, 1)^{card(M)}$ as closed bounded subspaces. The latter space is uniformly homeomorphic with $pr - \lim_k (\mathbf{S}_{\mathbf{p}^{\mathsf{k}}})^{M_k}$, which is compact by the Tychonoff theorem 3.2.4 [6]. Since $C^0(M, N) \neq C^t(M, N)$ for $t > 0$, then $C_w(M, N)$ and $C^t(M, N)$ are different algebraically.

2.6. Let $Diff_w(M) := pr - \lim_k Hom(M_k)$ be supplied with the uniformity inherited from $C_w(M, M)$. The group $Diff_w(M)$ is called the p-adic compactification of $Diff^t(M)$. The following theorem shows that this terminology is justified.

Theorem. *$Diff_w(M)$ is the compact topological group and it is the compactification of $Diff^t(M)$ in the weak topology. If $t > 0$, then $Diff^t(M)$ does not coincide with $Diff_w(M)$.*

Proof. Since $Diff^t(M) \subset C^t(M, M)$, then $Diff^t(M)$ has the cor-

responding embedding into $C_w(M,M)$. Since $C_w(M,M)$ is compact and $Hom(M)$ is a closed subset in $C_w(M,M)$, then due to Corollary 2.5 $Hom(M)\cap C_w(M,M) = Diff_w(M)$ is compact. The space $C^t(M,M)$ is dense in $C^0(M,M)$, consequently, $Diff^t(M)$ is dense in $Diff_w(M)$. If $t>0$, then $Diff^t(M)\ne Hom(M)$, hence $Diff^t(M)$ and $Diff_w(M)$ do not coincide algebraically. It remains to verify, that $Diff_w(M)$ is the topological group in its weak topology. If $f,g\in C^t(M,N)$, then $\pi_k(\bar Q_m(g(x))) = \bar Q_{m,k}(g_k(x(k))$, consequently, $\pi_k(f\circ g) = \sum_{l,m}\pi_k(f^l_m)\bar Q_{m,k}(g_k(x(k))e_l$ and inevitably $(f\circ g)_k = f_k\circ g_k$. On the other hand $\pi_k(x)=x(k)$, hence $\pi_k(id(x)) = id_k(x(k))$, where $id(x)=x$ for each $x\in M$. Therefore, for $f=g^{-1}$ we have $(f\circ g)_k = f_k\circ g_k = id_k$, hence $\pi_k(g^{-1}) = g_k^{-1}$. The associativity of the composition $(f_k\circ g_k)\circ h_k = f_k\circ(g_k\circ h_k)$ of all functions $f_k, g_k, h_k\in Hom(M_k)$ together with others properties given above means, that $Diff_w(M)$ is the algebraic group, since $f = pr-\lim_k f_k$, $g = pr-\lim_k g_k$ and $h = pr-\lim_k h_k$ also satisfy the associativity axiom, each f has the inverse element $f^{-1}(f(x)) = id$ and $e=id$ is the unit element. By the definition of the weak topology in $Diff_w(M)$ for each neighbourhood of $e=id$ in $Diff_w(M)$ there exists $k\in \mathbf{N}$ and a subset $W_k\subset Hom(M_k)$ such that $e_k\in\pi_k^{-1}(W_k)\subset W$. But $Hom(M_k)$ is discrete, hence there are $e_k\in V_k\subset Hom(M_k)$ and $e_k\in U_k\subset Hom(M_k)$ such that $V_kU_k\subset W_k$, hence there are neighbourhoods $e\in V\subset Diff_w(M)$ and $e\in U\subset Diff_w(M)$ such that $VU\subset W$, where $V=\pi_k^{-1}(V)$, $U=\pi_k^{-1}(U)$ and $VU = \{h: h=f\circ g, f\in V, g\in U\}$. If W' is a neighbourhood of f^{-1}, then $V := W'f^{-1}$ is the neighbourhood of e and there exists $k\in\mathbf{N}$ such that $\pi_k^{-1}(e_k) =: U\subset V^{-1}$, since $e_k^{-1} = e_k$ and π_k is the homomorphism. Therefore, $fU := W$ is the neighbourhood of f such that $W^{-1}\subset W'$, that demonstrates the continuity of the inversion operation $f\mapsto f^{-1}$.

2.7. Notes. Each projection $\pi_k : C^t(M,\mathbf{K}^{\mathsf{n}})\to(\mathbf{K}^{\mathsf{n}}_{\mathbf{k}})^{M_k}$ produces the quotient metric ρ_k in the $\mathbf{K_k}$-module $(\mathbf{K}^{\mathsf{n}}_{\mathbf{k}})^{M_k}$ such that $\rho_k(f_k,g_k) := inf_{z,\pi_k(z)=0}\|f-g+z\|_{C^t(M,\mathbf{K}^{\mathsf{n}})}$, where $\mathbf{K_k} := \mathbf{K}/B(\mathbf{K},0,p^{-k})$ is the quotient ring and π_k is induced by such quotient mapping from $\mathbf{K}$ onto $\mathbf{K_k}$. If $C^t(M,\mathbf{K}^{\mathsf{n}})$ embed into $\prod_k\pi_k(C^t(M,\mathbf{K}^{\mathsf{n}}))$ and supply the latter space with the box topology given by the following norm $\|f-g\|' := \sup_k\rho_k(f_k,g_k)$, then it produces the uniformity in $C^t(M,\mathbf{K}^{\mathsf{n}})$ equivalent with the initial one.

Theorem 2.6 means that the p-adic completion $Diff_w(M)$ is the profinite group, that is, it is projective limit of finite groups $Hom(M_k)$. If the compact manifold M is decomposed into the disjoint union $M=\bigcup_i B(\mathbf{K}^{\mathsf{m}},x_i,r_i)$ of clopen balls, then orders of the latter groups are divisible by $(p^a)!$, where

$a = \sum_i l_i$, $l_i = k - \max_l\{l : p^l \le r_i\}$, $x_i \in B(\mathbf{K}^{\mathsf{m}}, 0, 1)$, $0 < r_i \le 1$, since $card(M_k)$ is divisible by p^a. Then the representations of symmetric groups known from the classical works of A. Young and H. Weyl [8, 21] with the help of the projective limit decompositions produce finite-dimensional representations of the diffeomorphism groups.

3 p-Adic Completion of Loop Groups.

3.1. Let as in §2.1 $\bar{M}$ and N be two compact manifolds and $Diff_0^t(\bar{M})$ be a subgroup in $Diff^t(\bar{M})$ of all elements $\psi \in Diff^t(\bar{M})$ such that $\psi(s_0) = s_0$, where s_0 is a marked point in $\bar{M}$. We denote shortly by $C_0^t(M, N)$ a subspace in $C^t(\bar{M}, N)$ of all elements $f \in C^t(\bar{M}, N)$ such that $\lim_{|\zeta_1|+...+|\zeta_n| \to 0} \bar{\Phi}^v(f - w_0)(s_0; h_1, ..., h_n; \zeta_1, ..., \zeta_n) = 0$ for each $v \in \{0, 1, ..., [t], t\}$, $n = [v] + sign\{v\}$, where $M = \bar{M} \setminus s_0$ and $w_0(\bar{M}) = \{y_0\}$ (see §2.6 [11]).

Theorem. *Let $\Omega_\xi(M, N)$ be commutative loop monoids, then the quotient mappings π_k induce the corresponding inverse sequence $\{\Omega(M_k, N_k) : k \in \mathbf{N}\}$ such that $\Omega^w(M, N) := pr - \lim_k \Omega(M_k, N_k)$ is the commutative compact topological monoid, where $\pi_k : \Omega_\xi(M, N) \to \Omega(M_k, N_k)$, $\pi_k^l : \Omega(M_l, N_l) \to \Omega(M_k, N_k)$ are surjective mappings for each $l \ge k$, $\Omega(M_k, N_k) = \{f_k : f_k \in N_k^{M_k}, f_k(s_{0,k}) = y_{0,k}\}/K_{\xi,k}$, $K_{\xi,k}$ is an equivalence relation induced by an equivalence relation K_ξ. Moreover, $\Omega^w(M, N)$ is a compactification of $\Omega_\xi(M, N)$.*

Proof. In view of Corollary 2.3 $\pi_k(C_0^\xi(M, N))$ is isomorphic with $\{f_k : f_k \in N_k^{M_k}, f_k(s_{0,k}) = y_{0,k}\}$, where the quotient mapping is denoted by π_k both for M and N, since it is induced by the same ring homomorphism $\pi_k : \mathbf{K} \to \mathbf{K}/B(\mathbf{K}, 0, 1)$, $s_{0,k} := \pi_k(s_0)$ and $y_{0,k} := \pi_k(y_0)$. Then $\pi_k(Diff_0^t(M))$ is isomorphic with $Hom_0(M_k) := \{\psi_k : \psi_k \in Hom(M_k), \psi_k(s_{0,k}) = s_{0,k}\}$. All of this is also applicable with the corresponding changes to classes of smoothness C^ξ (or $C(\xi)$ in the notation of [11]), where $\xi = (t, s)$. If f and g are two K_ξ-equivalent elements in $C_0^\xi(M, N)$, that is, there are sequences f_n and g_n in $C_0^\xi(M, N)$ converging to f and g respectively and also a sequence $\psi_n \in Diff_0^\xi(M)$ such that $f_n(x) = g_n(\psi_n(x))$ for each $x \in M$, then $\pi_k(f_n) =: f_{n,k}$ and $g_{n,k} := \pi_k(g_n)$ converge to $\pi_k(f)$ and $\pi_k(g)$ respectively and also $\psi_{n,k} := \pi_k(\psi_n) \in Hom_0(M_k)$. From the equality $f_{n,k}(x(k)) = g_{n,k}(\psi_{n,k}(x(k)))$ for each $n \in \mathbf{N}$ and $x(k) \in M_k$ it follows, that the equivalence relation K_ξ induces the corresponding equiva-

lence relation $K_{\xi,k}$ in $\pi_k(C_0^t(M,N))$ such that classes $< \pi_k(f) >_{K,\xi,k}$ of $K_{\xi,k}$-equivalent elements are closed. Each element $f_k \in \pi_k(C_0^\xi(M,N))$ is characterized by the equality $f_k(s_{0,k}) = y_{0,k}$. Each $\Omega(M_k,N_k)$ is the finite discrete set, since each $N_k^{M_k}$ is the finite discrete set. This induces the quotient mapping $\pi_k : \Omega_t(M,N) \to \Omega(M_k,N_k)$ and surjective mappings $\pi_k^l : \Omega(M_l,N_l) \to \Omega(M_k,N_k)$ for each $l \ge k$ that produces the inverse sequence of finite discerete spaces, hence the limit of the inverse sequence is compact and totally disconnected. It remains to verify that $\Omega^w(M,N)$ is the commutative topoogical monoid with the unit element and the cancellation property.

From the equality $M = \bar{M} \setminus \{s_0\}$, it follows that $M_k = \bar{M}_k$, since for each $k \in \mathbf{N}$ there exists $x \in M$ such that $x + B(\mathbf{K}^{\mathsf{m}},0,p^{-k}) \ni s_0$. Moreover, M_k and N_k are finite dicrete spaces. Then $\pi_k(M \vee M) = M_k \vee M_k$, where $A \vee B := A \times \{b_0\} \cup \{a_0\} \times B \subset A \times B$ is the wedge product of pointed spaces (A,a_0) and (B,b_0), A and B are sets with marked points $a_0 \in A$ and $b_0 \in B$. The composition operation is defined on threads $\{< f_k >_{K,\xi,k}: k \in \mathbf{N}\}$ of the inverse sequence in the following way. There was fixed a C^∞-diffeomorphism $\chi : M \vee M \to M$. Let $x \in M$, then $\pi_k(x) \in M_k$ and $\chi^{-1}(U) \in M \vee M$, where $U := \pi_k^{-1}(x + B(\mathbf{K},0,p^{-k}) \cap M$. On the other hand $\chi^{-1}(U)$ is a disjoint union of balls of radius p^{-2k} in $B(\mathbf{K^{2m}},0,1)$, hence there is defined a surjective mapping $\chi_k : M_{2k} \vee M_{2k} \to M_k$ induced by χ, π_k and π_{2k} such that $\chi_k(\chi^{-1}(U)) = \pi_k(x)$. If f and $g \in C^\xi(M,N)$, then $f \vee g \in C^\xi((M \vee M),N)$ and $\chi(f \vee g) \in C^\xi(M,N)$ as in §2.6 [11]. Hence $\chi_k(f_{2k} \vee g_{2k}) \in C^\xi(M_k,N_k)$ and inevitably $\chi_k(< f_{2k} \vee g_{2k} >_{K,\xi,2k}) = \chi_k(< f_{2k} >_{K,\xi,2k} \vee < g_{2k} >_{K,\xi,2k}) \in \Omega(M_k,N_k)$.

There exists a one to one correspondence between elements $f \in C_w(\bar{M},N)$ and $\{f_k : k\} \in \{N_k^{M_k} : k\}$. Therefore, $pr-\lim_k \Omega(M_k,N_k)$ algebraically is the commutative monoid with the cancellation property. Let U be a neighbourhood of e in $\Omega^w(M,N)$, then there exists $U_k = \pi_k^{-1}(V_k)$ such that V_k is open in $\Omega(M_k,N_k)$, $e \in U_k$ and $U_k \subset U$. On the other hand there exists $U_{2k} = \pi_{2k}^{-1}(V_{2k})$ such that V_{2k} is open in $\Omega(M_{2k},N_{2k})$, $e \in U_{2k}$ and $U_{2k} + U_{2k} \subset U_k$. Therefore, $(f + U_{2k}) + (g + U_{2k}) \subset f + g + U_k \subset f + g + U$ for each $f, g \in \Omega^w(M,N)$, consequently, the composition in $\Omega^w(M,N)$ is continuous. Since $C_0^\xi(M,N)$ is dense in $C_{0,w}(\bar{M},N)$, then $\Omega_\xi(M,N)$ is dense in $\Omega^w(M,N)$.

3.2. Note. The compactificiation of $\Omega_\xi(M,N)$ given above is not

unique. Another compactification is given below. The second is larger than the first one. Using the Grothendieck construction we get a compactification $L^w(M,N) = \bar{F}/\bar{B}$ of a loop group $L_\xi(M,N)$, where $\bar{F}$ is a closure in $(\Omega^w(M,N))^{\mathbf{Z}}$ of a free commutative group F generated by $\Omega^w(M,N)$ and $\bar{B}$ is a closure of a subgroup B generated by all elements $[a+b]-[a]-[b]$, since the product of compact spaces is compact by the Tychonoff theorem.

3.3. Let now $s_0 = 0$ and $y_0 = 0$ be two marked points in the compact manifolds $\bar{M}$ and N embedded into $\mathbf{K}^{\mathsf{m}}$ and $\mathbf{K}^{\mathsf{n}}$ respectively. There is defined the following C^∞-diffeomorphism $inv : (\mathbf{K}^{\mathsf{m}})' \to (\mathbf{K}^{\mathsf{m}})'$ for $(\mathbf{K}^{\mathsf{m}})' := \mathbf{K}^{\mathsf{m}} \setminus \{x :$ there exists j with $x_j = 0\}$ such that $inv(x_1, ..., x_{\mathsf{m}}) = (x_1^{-1}, ..., x_{\mathsf{m}}^{-1})$. Let $M' = M \cap (\mathbf{K}^{\mathsf{m}})'$, then $inv(M')$ is locally compact and unbounded in $\mathbf{K}^{\mathsf{m}}$, consequently, $\pi_k(inv(M')) = (inv(M'))_k$ is a discrete infinite subset in $\mathbf{K}^{\mathsf{m}}_{\mathbf{k}}$ for each $k \in \mathbf{N}$. Analogously $\pi_k(inv(M' \vee M')) = (inv(M' \vee M'))_k \subset \mathbf{K}^{2\mathsf{m}}_{\mathbf{k}}$. There exists a C^∞-diffeomorphism $\chi : M \vee M \to M$ such that $inv \circ \chi \circ inv$ is the C^∞-diffeomorphism of $inv(M' \vee M')$ with $inv(M')$ and it induces bijective mappings χ_k of $inv((inv(M' \vee M'))_k)$ with $inv((inv(M'))_k)$ for each $k \in \mathbf{N}$ such that $\hat{\pi}^l_k \circ \chi_l = \chi_k$ for each $l \geq k$, where $\hat{\pi}^l_k := inv \circ \pi^l_k \circ inv$. This produces inverse sequences of discrete spaces $inv((inv(M'))_k) =: \hat{M}_k$, $inv((inv(M' \vee M'))_k) = \hat{M}_k \vee \hat{M}_k$ and their bijections χ_k such that $pr - \lim_k \hat{M}_k$ is homeomorphic with M' and $pr - \lim_k \chi_k$ is equal to χ up to the homeomorphism, since $pr - \lim_k \mathbf{K}^{\mathsf{m}}_{\mathbf{k}} = \mathbf{K}^{\mathsf{m}}$ (see also about admissible modifications and polyhedral expansions in [12]). If $\psi \in Diff^\xi_0(\bar{M})$, then $\hat{\psi} \in Diff^\xi(\hat{M})$. Let $J_{f,k} := \{h_k : h_k = f_k \circ \psi_k, \psi_k \in Hom(\hat{M}_k), \psi_k(s_{0,k}) = s_{0,k}\}$ for $f_k \in N_k^{\hat{M}_k}$ with $\lim_{x\to 0} f_k(x) = 0$, then $J_{f,k}$ is closed and $\hat{\pi}_k(< f >_{K,\xi}) \subset J_{f,k}$. Therefore, g_k and f_k are $\hat{K}_{\xi,k}$-equivalent if and only if there exists $\psi_k \in Hom(\hat{M}_k)$ such that $\psi_k(s_{0,k}) = s_{0,k}$ and $g_k(x) = f_k(\psi_k(x))$ for each $x \in \hat{M}_k$. Let $\Omega(\hat{M}_k, N_k) := \hat{\pi}_k(\Omega_\xi(M,N))$.

Theorem. *The set of $\Omega(\hat{M}_k, N_k)$ forms an inverse sequence $S = \{\Omega(\hat{M}_k, N_k); \hat{\pi}^l_k; k \in \mathbf{N}\}$ such that $pr - \lim S =: \Omega^{i,w}(M,N)$ is an associative topological loop monoid with the cancellation property and the unit element e. There exists an embedding of $\Omega_\xi(M,N)$ into $\Omega^{i,w}(M,N)$ such that $\Omega_\xi(M,N)$ is dense in $\Omega^{i,w}(M,N)$.*

Proof. Let U'_i be an analytic disjoint atlas of $inv(M')$, $f \in C^\xi(inv(M'), \mathbf{K})$, $\psi \in Diff^\xi(inv(M'))$, then each restriction $f|_{U'_i}$ has the form $f|_{U'_i}(x) = \sum_m f_{i,m}\bar{Q}_{i,m}(x)$ for each $x \in U'_i$, where $\bar{Q}_{i,m}$ are basic Amice polynomials for U'_i, $f_{i,m} \in \mathbf{K}$. Therefore f is a combination $f = \nabla_i f|_{U'_i}$, hence

$\hat{\pi}_k(f\circ\psi(x)) = \sum_m[\hat{\pi}_k(f_{i,m})\nabla_{(i,\psi_k(x(k))\in\hat{\pi}_k(U'_k)}\bar{Q}_{i,m,k}(\psi_k(x(k)))]$ and inevitably $\hat{\pi}_k((f\circ\psi)(x)) = f_k\circ\psi_k(x(k))$, where $\bar{Q}_{i,m,k} := \hat{\pi}_k(\bar{Q}_{i,m})$, $x\in inv(M')$ and $x(k) = \hat{\pi}_k(x)$.

As in §2.6.2 [11] we choose an infinite atlas $At'(M) := \{(U'_j, \phi'_j) : j\in \mathbf{N}\}$ such that $\phi'_j : U'_j \to B(X, y'_j, r'_j)$ are homeomorphisms, $\lim_{k\to\infty} r'_{j(k)} = 0$, $\lim_{k\to\infty} y'_{j(k)} = 0$ for an infinite sequence $\{j(k)\in\mathbf{N} : k\in\mathbf{N}\}$ such that $cl_{\bar{M}}[\bigcup_{k=1}^{\infty} U'_{j(k)}]$ is a clopen neighbourhood of zero in $\bar{M}$, where $cl_{\bar{M}}A$ denotes the closure of a subset A in $\bar{M}$. We take $|y'_{j(k)}| > r'_{j(k)}$ for each k, hence $inv(B(X, y'_j, r'_j)\cap X') = B(X, {y'}_j^{-1}, {r'}_j^{-1})\cap X'$ and $\bigcup_k inv(U'_{j(k)}\cap X')$ is open in X', where $X = \mathbf{K}^{\mathsf{m}}$. For an atlas $At'(M\vee M) := \{(W_l, \xi_l) : l\in\mathbf{N}\}$ with homeomorphisms $\xi_l : W_l \to B(X, z_l, a_l)$, $\lim_{k\to\infty} a_{l(k)} = 0$, $\lim_{k\to\infty} z_{l(k)} = 0$ for an infinite sequence $\{l(k)\in\mathbf{N} : k\in\mathbf{N}\}$ such that $cl_{\bar{M}\vee\bar{M}}[\bigcup_{k=1}^{\infty} W_{l(k)}]$ is a clopen neighbourhood of 0×0 in $\bar{M}\vee\bar{M}$ we also choose $|z_l| > a_l$ for each l, where $card(\mathbf{N}\setminus\{l(k) : k\in\mathbf{N}\}) = card(\mathbf{N}\setminus\{j(k) : k\in\mathbf{N}\})$. Then we take $\chi(W_{l(k)}) = U'_{j(k)}$ for each $k\in\mathbf{N}$ and $\chi(W_l) = U'_{\kappa(l)}$ for each $l\in(\mathbf{N}\setminus\{l(k) : k\in\mathbf{N}\})$, where $\kappa : (\mathbf{N}\setminus\{l(k) : k\in\mathbf{N}\}) \to (\mathbf{N}\setminus\{j(k) : k\in\mathbf{N}\})$ is a bijective mapping such that $p^{-1}\le r'_{j(k)}/a_{l(k)}\le p$ for each k and $p^{-1}\le r'_{\kappa(l)}/a_l\le p$ for each $l\in(\mathbf{N}\setminus\{l(k) : k\in\mathbf{N}\})$. We can choose the locally affine mapping χ such that $\Phi^n\chi = 0$ for each $n\ge 2$ and $B(X', {y'}_l^{-1}, {r'}_l^{-1})$ are diffeomorphic with $inv(U'_l\cap X')$ and $B(X'\vee X', z_l^{-1}, a_l^{-1})$ are diffeomorphic with $inv(W_l\cap(X'\vee X'))$.

This induces the diffeomorphisms $\hat{\chi} := inv\circ\chi\circ inv : \hat{M}\vee\hat{M}\to\hat{M}$ and $\hat{\chi}^* : C^{\xi}_0((\hat{M}\vee\hat{M}, \infty\times\infty), (N, y_0)) \to C^{\xi}_0((\hat{M}, \infty), (N, y_0))$, since each $\Phi^n(f\vee g)(\hat{\chi}^{-1})$ has an expression through $\Phi^l(f\vee g)$ and $\Phi^j(\hat{\chi}^{-1})$ with $l, j\le n$ and n subordinated to ξ, where $\hat{M} := inv(M')$ and conditions defining the subspace $C^{\xi}_0((\hat{M}, \infty), (N, y_0))$ differ from that of $C^{\xi}_0((M, s_0), (N, y_0))$ by substitution of $\lim_{x\to s_0}$ on $\lim_{|x|\to\infty}$. Then $\lim_{|x|\to\infty}|\hat{\chi}(x)| = \infty$, consequenlty, there exists $k_0\in\mathbf{N}$ such that $\hat{\chi}_k : \hat{M}_k\vee\hat{M}_k\to\hat{M}_k$ are bijections for each $k\ge k_0$, where $\hat{\chi}_k := \hat{\pi}_k\circ\hat{\chi}$. If $\psi\in Diff^{\xi}(\bar{M})$ and $\psi(0) = 0$, then $\lim_{|x|\to\infty}\hat{\psi}(x) = \infty$ and $\lim_{|x|\to\infty}\hat{\psi}^{-1}(x) = \infty$. Then considering $\hat{\psi}_k$ we get an equivalence relation $K_{\xi,k}$ in $\{f_k : f_k\in N_k^{\hat{M}_k}, \lim_{|x|\to\infty} f_k(x) = 0\}$ induced by K_{ξ}, where $\hat{M}_k$ is supplied with the quotient norm induced from the space X, since $X'\subset X$, $x\in\hat{M}_k$. Let J_k denotes the quotient mapping corresponding to $K_{\xi,k}$. Therefore analogously to §2.6 [11] we get, that $\Omega(\hat{M}_k, N_k)$ are commuative monoids with the cancellation property and the unit elements e_k, since $\Omega(\hat{M}_k, N_k) = \{f_k : f_k\in C^0(\hat{M}_k, N_k), \lim_{|x|\to\infty} f_k(x) = 0\}/\hat{K}_{\xi,k}$ and

mappings $\hat{\pi}^l_k : (\mathbf{K}^{\mathsf{m}})'_l \to (\mathbf{K}^{\mathsf{m}})'_k$ and mappings $\pi^l_k : \mathbf{K}^{\mathsf{n}}{}_l \to \mathbf{K}^{\mathsf{n}}{}_k$ induce mappings $\hat{\pi}^l_k : \Omega(\hat{M}_l, N_l) \to \Omega(\hat{M}_k, N_k)$ for each $l \ge k$. Let the topology in $\{f_k : f_k \in C^0(\hat{M}_k, N_k), \lim_{|x|\to\infty} f_k(x) = 0\}$ be induced from the Tychonoff product topology in $N_k^{\hat{M}_k}$ and $\Omega(\hat{M}_k, N_k)$ be in the quotient topology. The space $N_k^{\hat{M}_k}$ is metrizable by the Baire metric $\rho(x,y) := p^{-j}$, where $j = \min\{i : x_i \ne y_i, x_1 = y_1, ..., x_{i-1} = y_{i-1}\}$, $x = (x_l : x_l \in N_k, l \in \mathbf{N})$, $\hat{M}_k$ as enumerated as $\mathbf{N}$. Therefore, $\Omega(\hat{M}_k, N_k)$ is metrizable and the mapping $(f_k, g_k) \to f_k \vee g_k$ is continuous, hence the mapping $(J_k(f_k), J_k(g_k)) \to J_k(f_k) \circ J_k(g_k)$ is also continuous. Then $J_k(w_{0,k})$ is the unit element, where $w_{0,k}(\hat{M}_k) = 0$. Hence $\Omega^{i,w}(M,N)$ is the commutative monoid with the cancellation property and the unit element. Certainly $\prod_k \Omega(\hat{M}_k, N_k)$ is the topological monoid and $pr - \lim S$ is a closed in it topological totally disconnected monoid. For each $f \in C^{\xi}_0(M,N)$ there exists an inverse sequence $\{f_k : f_k = \hat{\pi}_k(f), k \in \mathbf{N}\}$ such that $f(x) = pr - \lim_k f_k(x(k))$ for each $x \in M'$, but M' is dense in M. Therefore there exists an embedding $\Omega^{\xi}(M,N) \hookrightarrow \Omega^{i,w}(M,N)$. Since $C^{\xi}(M,N)$ is dense in $C^0_0(M,N)$, then $\Omega^{\xi}(M,N)$ is dense in $\Omega^{i,w}(M,N)$.

3.4. Corollary. *The inverse sequence of loop monoids induces the inverse sequence of loop groups $S_L := \{L(\hat{M}_k, N_k); \hat{\pi}^l_k; \mathbf{N}\}$. Its projective limit $L^{i,w}(M,N) := pr - \lim S_L$ is a commutative topological totally disconnected group and $L_{\xi}(M,N)$ has an embedding in it as a dense subgroup.*

Proof. Due to Grothendieck construction the inversion operation $f_k \mapsto f_k^{-1}$ is continous in $L(\hat{M}_k, N_k)$ and homomorphisms $\hat{\pi}^l_k$ and $\hat{\pi}_k$ have continous extensions from loop submonoids onto loop groups $L(\hat{M}_k, N_k)$. Each monoid $\Omega(\hat{M}_k, N_k)$ is totally disconnected, since $N_k^{\hat{M}_k}$ is totally disconnected and $\Omega(\hat{M}_k, N_k)$ is supplied with the quotient ultrametric, hence the free Abelian group F_k generated by $\Omega(\hat{M}_k, N_k)$ is also totally disconnected and ultramertizable, consequenlty, $L(\hat{M}_k, N_k)$ is ultrametrizable. Evidently their inverse limit is also ultrametrizable and the equivalent ultrametric can be chosen with values in $\tilde{\Gamma}_{\mathbf{K}} := \{|z| : z \in \mathbf{K}\}$, where $\tilde{\Gamma}_{\mathbf{K}} \cap (0,\infty)$ is discrete in $(0,\infty) := \{x : 0 < x < \infty, x \in \mathbf{R}\}$. Then the projective limit (that is, weak) topology of $L^{i,w}(M,N)$ is induced by the weak topology of $C^0(M,\mathbf{K})$. When M and N are non-trivial, then certainly this weak topology is strictly weaker, than that of $L_0(M,N)$.

3.5 Theorem. *For each prime number p the loop group $L_{\xi}(M,N)$ in its weak topology inherited from $L^{i,w}(M,N)$ has the p-adic completion isomorphic with $\mathbf{Z_p}^{\aleph_0}$.*

Proof. If $\mathbf{P}$ is an extension of the field $\mathbf{K}$, then the projective ring homomorphism $\pi_k : \mathbf{K} \to \mathbf{K_k}$ induces the ring homomorphism $\pi_k : \mathbf{P} \to \mathbf{P_k}$, then $\hat{\pi}_k(\bar{\Phi}^v(f(x;h_1,...,h_n;\zeta_1,...,\zeta_n)) = \bar{\Phi}^v f_k(x(k);h_1(k),...,h_n(k);\zeta_1(k),...,\zeta_n(k))$, where $n = [v] + sign\{v\}$, $j_b : \mathbf{K} \to \mathbf{P}$, $\bar{\Phi}^v f_k$ is defined for the field of fractions generated by $\mathbf{P_k}$ (see also §§2.1-2.6 [11]). Then the condition

$$\lim_{|x|\to\infty} \bar{\Phi}^v f(x;h_1,...,h_n;\zeta_1,...,\zeta_n) = 0$$

implies the condition

$$\lim_{|x(k)|\to\infty} \bar{\Phi}^v f_k(x(k);h_1(k),...,h_n(k);\zeta_1(k),...,\zeta_n(k)) = 0.$$

Therefore, $supp(f_k) := \hat{M}_k^f := \{x(k) : f_k(x(k)) \ne 0\}$ is a finite subset of the discrete space $\hat{M}_k$ for each $k \in \mathbf{N}$. Then evidently, $\hat{\pi}_k(< g >_{K,\xi})$ is a closed subset in $N_k^{\hat{M}_k}$ for each $g \in C_0^\xi((\hat{M},\infty),(N,0))$, since for each limit point f_k of $\hat{\pi}_k(< g >_{K,\xi})$ its support is the finite subset in $\hat{M}_k$. Let k_0 be such that $N_{k_0} \ne \{0\}$, then this is also true for each $k \ge k_0$. If $f_k \notin \hat{\pi}_k(< w_0 >_{K,\xi})$ and $k \ge k_0$, then $f_k^{\vee n} \notin \hat{\pi}_k(< w_0 >_{K,\xi})$ for each $n \in \mathbf{N}$, where $f_k^{\vee n} := f_k \vee ... \vee f_k$ denotes the n-times wedge product, since $\|f^{\vee n}\|_{C^\xi} \ge \|f\|_{C^\xi} > 0$ and $\|f_k^{\vee n}\|_{C(\mathbf{K_k^m},\mathbf{K^n})} \ge \|f\|_{C(\mathbf{K_k^m},\mathbf{K^n})} > 0$, where $C(\mathbf{K_k^m},\mathbf{K^n}) = \pi_k(C^\xi(\mathbf{K^m},\mathbf{K^n}))$ is the quotient module over the ring $\mathbf{K_k}$. Each $\hat{\pi}_k(< f >_{K,\xi})$ can be presented as the following composition $v_1b_1 + ... + v_lb_l$ in the additive group $L(\hat{M}_k, N_k)$, where each b_i corresponds to $\hat{\pi}_k(< g_i >_{K,\xi})$ and the embedding of $\Omega(\hat{M}_k, N_k)$ into $L(\hat{M}_k, N_k)$, $v_i \in \{-1,0,1\}$, $l = card(\hat{M}_k^f)$, $\hat{M}_k^{g_i}$ are singletons for each $i = 1,...,l$. Using the group $Hom_0(N_k)$ we get that $L(\hat{M}_k, N_k)$ is isomorphic with $\mathbf{Z}^{n_k}$, where $n_k = card(N_k) > 1$. In view of Corollary 3.4 $L_\xi(M,N)$ has the p-adic completion isomorphic with $\mathbf{Z}_p^{\aleph_0}$, since $\mathbf{Z}$ is dense in $\mathbf{Z_p}$ and $pr - \lim_k \mathbf{Z}^{n_k} = \mathbf{Z}^{\aleph_0}$.

3.6. Note. Using quotient mappings $\eta_{p,s} : \mathbf{Z} \to \mathbf{Z}/p^s\mathbf{Z}$ we get that $L_\xi(M,N)^{\aleph_0}$ has the compactification equal to $\prod_{p\in\mathsf{P}} \mathbf{Z_p}^{\aleph_0}$, where P denotes the set of all prime numbers $p > 1$, $s \in \mathbf{N}$. These compactifications produce characters of $L_\xi(M,N)$, since each compact Abelian group has only one-dimensional irreducible unitary representations [7]. On the other hand, there are irreducible continuous representations of compact groups in non-Archimedean Banach spaces [18]. Among them there are infinite-dimensional [5]. Moreover, in their initial topologies diffeomorphism and loop groups also have infinite-dimensional irreducible unitary representations [10, 11].

The problem about p-adic completions of diffeomorphism and loop groups of manifolds on non-Archimedean Banach spaces over local fields was formulated by B. Diarra after reading articles of S.V. Ludkovsky on such groups. Then S.V. Ludkovsky investigated this problem and all his results and proofs were thoroughly and helpfully discussed and were corrected with B. Diarra.

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